

# ANALYSIS THE $f_0(980)$ AND $a_0(980)$ MESONS AS FOUR-QUARK STATES WITH THE QCD SUM RULES

Zhi-Gang Wang<sup>1</sup> and Wei-Min Yang<sup>2</sup>

<sup>1</sup> Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

<sup>2</sup> Department of Modern Physics, University of Science and Technology of China, Hefei 230026, P. R. China

## Abstract

In this article, we take the point of view that the scalar mesons  $f_0(980)$  and  $a_0(980)$  are diquark-antidiquark states  $(qq)_3(\bar{q}\bar{q})_3$ , and devote to determine their masses in the framework of the QCD sum rules approach with the interpolating currents constructed from scalar-scalar type and pseudoscalar-pseudoscalar type diquark pairs respectively. The numerical results indicate that the scalar mesons  $f_0(980)$  and  $a_0(980)$  may have two possible diquark-antidiquark substructures.

PACS numbers: 12.38.Lg; 13.25.Jx; 14.40.Cs

**Key Words:**  $f_0(980)$ ,  $a_0(980)$ , QCD sum rules

## 1 Introduction

The light flavor scalar mesons present a remarkable exception for the constituent quark model and the structures of those mesons have not been unambiguously determined yet [1]. Experimentally, the strong overlaps with each other and the broad widths (for the  $f_0(980)$ ,  $a_0(980)$  et al, the widths are comparatively narrow) make their spectra cannot be approximated by the Breit-Wigner formula. The numerous candidates with the same quantum numbers  $J^{PC} = 0^{++}$  below  $2GeV$  can not be accommodated in one  $q\bar{q}$  nonet, some are supposed to be glueballs, molecules and multiquark states. The more elusive things are the constituent structures of the mesons  $f_0(980)$  and  $a_0(980)$  with almost the degenerate masses. In the naive quark model,  $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$  and  $f_0 = s\bar{s}$ ; while in the framework of four-quark models, the mesons  $f_0(980)$  and  $a_0(980)$  could either be compact objects i.e. nucleon-like bound states of quarks with symbolic quark structures  $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$  and  $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$  [2], or spatially extended objects i.e. deuteron-like bound states of hadrons, the  $f_0(980)$  and  $a_0(980)$  mesons are usually taken as  $K\bar{K}$  molecules [3]. The hadronic dressing mechanism takes the point of view that the mesons  $f_0(980)$  and  $a_0(980)$  have small  $q\bar{q}$  cores of typical  $q\bar{q}$  meson size, the strong couplings to the hadronic channels enrich the pure  $q\bar{q}$  states with other components and spend part (or most part) of their lifetime as virtual  $K\bar{K}$  states [4]. In the hybrid model, those mesons are four-quark states  $(qq)_3(\bar{q}\bar{q})_3$  in S-wave near the center, with some

---

<sup>1</sup>Corresponding author; E-mail, wangzgyiti@yahoo.com.cn.

constituent  $q\bar{q}$  in P-wave, but further out they rearrange into  $(q\bar{q})_1(q\bar{q})_1$  states and finally as meson-meson states [5]. All those interpretations have both outstanding advantages and obvious shortcomings in one or other ways.

There may exist two scalar nonets below  $1.7\text{GeV}$ . The attractive interactions of one gluon exchange favor the formation of diquarks in color antitriplet  $\bar{3}_c$ , flavor antitriplet  $\bar{3}_f$  and spin singlet  $1_s$ . The strong attractions between the states  $(qq)_{\bar{3}}$  and  $(\bar{q}\bar{q})_3$  in S-wave may result in a nonet manifested below  $1\text{GeV}$  while the conventional  $^3P_0$   $\bar{q}q$  nonet would have masses about  $1.2 - 1.6\text{GeV}$ . Taking the diquarks and antidiquarks as the basic constituents, keeping the effects of the  $s$  quark mass at the first order, the two isoscalars  $\bar{u}dud$  and  $\bar{s}s\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}$  mix ideally, the  $\bar{s}s\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}$  degenerate with the isovectors  $\bar{s}s\bar{d}u$ ,  $\bar{s}s\frac{\bar{u}u-\bar{d}d}{\sqrt{2}}$  and  $\bar{s}s\bar{u}d$  naturally. Comparing with the traditional  $\bar{q}q$  nonet mesons, the mass spectrum is inverted. The lightest state is the non-strange isosinglet  $(\bar{u}dud)$ , the heaviest are the degenerate isosinglet and isovectors with hidden  $\bar{s}s$  pairs while the four strange states lie in between [5, 6].

In this article, we take the point of view that the well confirmed  $f_0(980)$  and  $a_0(980)$  mesons are four-quark states  $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$  in the ideal mixing limit, and devote to determine the values of their masses  $m_{f_0}$  and  $m_{a_0}$  in the framework of the QCD sum rules approach [7, 8, 9]. Detailed studies about the other scalar four-quark states (the  $\kappa(800)$ s are not confirmed yet) and the mixing between the two isoscalars (the  $f_0(980)$  meson and the broad  $f_0(600)$  meson) will be our next work.

The article is arranged as follows: in section II, we obtain the QCD sum rules for the masses of the mesons  $f_0(980)$  and  $a_0(980)$ ; in section III, numerical results; section IV is reserved for conclusion.

## 2 Masses of the $f_0(980)$ and $a_0(980)$ mesons with the QCD Sum Rules

In the four-quark models, the structures of the scalar mesons  $f_0(980)$  and  $a_0(980)$  in the ideal mixing limit can be symbolically taken as [2, 5, 6]

$$f_0(980) = \frac{us\bar{u}\bar{s} + ds\bar{d}\bar{s}}{\sqrt{2}}, \quad a_0(980) = \frac{us\bar{u}\bar{s} - ds\bar{d}\bar{s}}{\sqrt{2}}. \quad (1)$$

The four-quark configurations of the  $J^{PC} = 0^{++}$  mesons can give a lot of satisfactory descriptions of the hadron phenomenon, for example, the mass degeneracy of the  $f_0(980)$  and  $a_0(980)$  mesons, the mass hierarchy pattern of the scalar nonet, the large radiative widths of the  $f_0(980)$  and  $a_0(980)$  mesons, the  $D_s^+(c\bar{s})$  to  $\pi^+\pi^+\pi^-$  decay.

In the following, we write down the interpolating currents for the scalar mesons

$f_0(980)$  and  $a_0(980)$  based on the four-quark model [8, 9],

$$J_{f_0}^A = \frac{\epsilon_{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_5 s_c)(\bar{u}_d \gamma_5 C \bar{s}_e^T) + (d_b^T C \gamma_5 s_c)(\bar{d}_d \gamma_5 C \bar{s}_e^T)], \quad (2)$$

$$J_{f_0}^B = \frac{\epsilon_{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C s_c)(\bar{u}_d C \bar{s}_e^T) + (d_b^T C s_c)(\bar{d}_d C \bar{s}_e^T)], \quad (3)$$

$$J_{a_0}^A = \frac{\epsilon_{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_5 s_c)(\bar{u}_d \gamma_5 C \bar{s}_e^T) - (d_b^T C \gamma_5 s_c)(\bar{d}_d \gamma_5 C \bar{s}_e^T)], \quad (4)$$

$$J_{a_0}^B = \frac{\epsilon_{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C s_c)(\bar{u}_d C \bar{s}_e^T) - (d_b^T C s_c)(\bar{d}_d C \bar{s}_e^T)], \quad (5)$$

where  $a, b, c, \dots$  are color indices and  $C$  is the charge conjugation matrix. The constituents  $S^a(x) = \epsilon^{abc} u_b^T(x) C \gamma_5 s_c(x)$  and  $P^a(x) = \epsilon^{abc} u_b^T(x) C s_c(x)$  represent the scalar  $J^P = 0^+$  and the pseudoscalar  $J^P = 0^-$   $us$  diquarks respectively. They both belong to the antitriplet  $\bar{3}$  representation of the color  $SU(3)$  group and can cluster together to form  $S^a - \bar{S}^a$  type and  $P^a - \bar{P}^a$  type diquarks pairs to give the correct spin and parity for the scalar mesons  $J^P = 0^+$ . The scalar diquarks correspond to the  $^1S_0$  states of  $us$  and  $ds$  diquark systems. The one gluon exchange force and the instanton induced force can lead to significant attractions between the quarks in the  $0^+$  channels [10]. The pseudoscalar diquarks do not have nonrelativistic limit, can be taken as the  $^3P_0$  states.

The calculation of the  $a_0(980)$  meson as a four-quark state in the QCD sum rules approach was done originally for the decay constant and the hadronic coupling constants with the interpolating currents  $J_{a_0}^1$  and  $J_{a_0}^2$  [11, 12],

$$\begin{aligned} J_{f_0(a_0)}^1 &= \Sigma_{\Gamma=1, \pm\gamma_5} \bar{s} \Gamma s \frac{\bar{u} \Gamma u \pm \bar{d} \Gamma d}{\sqrt{2}}, \\ J_{f_0(a_0)}^2 &= \Sigma_{\Gamma=1, \pm\gamma_5} \bar{s} \Gamma \frac{\lambda^a}{2} s \frac{\bar{u} \Gamma \frac{\lambda^a}{2} u \pm \bar{d} \Gamma \frac{\lambda^a}{2} d}{\sqrt{2}}, \end{aligned} \quad (6)$$

where the  $\lambda^a$  is the  $SU(3)$  Gell-Mann matrix. Perform Fierz transformation both in the Dirac spinor and color space, for example, we can obtain

$$\begin{aligned} J_{f_0}^2 &\propto C_A J_{f_0}^A + C_B J_{f_0}^B + C_C \frac{\epsilon^{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_\mu s_c)(\bar{u}_d \gamma^\mu C \bar{s}_e^T) + (d_b^T C \gamma_\mu s_c)(\bar{d}_d \gamma^\mu C \bar{s}_e^T)] \\ &\quad + C_D \frac{\epsilon^{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_\mu \gamma_5 s_c)(\bar{u}_d \gamma^\mu \gamma_5 C \bar{s}_e^T) + (d_b^T C \gamma_\mu \gamma_5 s_c)(\bar{d}_d \gamma^\mu \gamma_5 C \bar{s}_e^T)] \dots, \\ J_{a_0}^2 &\propto C_A J_{a_0}^A + C_B J_{a_0}^B + C_C \frac{\epsilon^{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_\mu s_c)(\bar{u}_d \gamma^\mu C \bar{s}_e^T) - (d_b^T C \gamma_\mu s_c)(\bar{d}_d \gamma^\mu C \bar{s}_e^T)] \\ &\quad + C_D \frac{\epsilon^{abc}\epsilon_{ade}}{\sqrt{2}} [(u_b^T C \gamma_\mu \gamma_5 s_c)(\bar{u}_d \gamma^\mu \gamma_5 C \bar{s}_e^T) - (d_b^T C \gamma_\mu \gamma_5 s_c)(\bar{d}_d \gamma^\mu \gamma_5 C \bar{s}_e^T)] \dots \quad (7) \end{aligned}$$

Here  $C_A, C_B, C_C$  and  $C_D$  are coefficients which are not shown explicitly for simplicity. In the color superconductivity theory, the one gluon exchange induced Nambu–Jona-Lasinio like Models will also lead to the  $S^a - \bar{S}^a$  type and  $P^a - \bar{P}^a$  type diquark

pairs [13],

$$G\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\bar{q}\gamma_\mu\frac{\lambda^a}{2}q \propto C_AS^a\bar{S}^a + C_BP^a\bar{P}^a + \dots. \quad (8)$$

So we can take the point of view that the lowest lying scalar mesons are  $S$ -wave bound states of diquark-antidiquark pairs of  $S^a - \bar{S}^a$  type and  $P^a - \bar{P}^a$  type.

In this article, we investigate the masses of the scalar mesons  $f_0(980)$  and  $a_0(980)$  with two interpolating currents respectively and choose the following two-point correlation functions,

$$\Pi_S^i(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T [J_S^i(x) J_S^{i\dagger}(0)] | 0 \rangle. \quad (9)$$

Here the current  $J_S^i$  denotes  $J_{f_0}^A$ ,  $J_{f_0}^B$ ,  $J_{a_0}^A$  and  $J_{a_0}^B$ . According to the basic assumption of current-hadron duality in the QCD sum rules approach [7], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator  $J_S^i(x)$  into the correlation functions in Eq.(9) to obtain the hadronic representation. Isolating the ground state contributions from the pole terms of the mesons  $f_0(980)$  and  $a_0(980)$ , we get the result,

$$\Pi_S^i(p) = \frac{2f_S^{i2}m_s^{i8}}{m_S^{i2} - p^2} + \dots, \quad (10)$$

where the following definitions have been used,

$$\langle 0 | J_S^i | S \rangle = \sqrt{2} f_S^i m_S^{i4}. \quad (11)$$

We have not shown the contributions from the higher resonances and continuum states explicitly for simplicity.

The calculation of operator product expansion in the deep Euclidean space-time region is straightforward and tedious, technical details are neglected for simplicity. In this article, we consider the vacuum condensates up to dimension six. Once the analytical results are obtained, then we can take the current-hadron dualities below the thresholds  $s_0$  and perform the Borel transformation with respect to the variable  $P^2 = -p^2$ , finally we obtain the following sum rules,

$$2f_{f_0(a_0)}^{A2} m_{f_0(a_0)}^{A8} e^{-\frac{m_{f_0(a_0)}^{A2}}{M^2}} = AA, \quad (12)$$

$$2f_{f_0(a_0)}^{B2} m_{f_0(a_0)}^{B8} e^{-\frac{m_{f_0(a_0)}^{B2}}{M^2}} = BB, \quad (13)$$

$$\begin{aligned} AA &= \int_{4m_s^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ \frac{s^4}{2^9 5! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma G q \rangle - \langle \bar{s}g_s \sigma G s \rangle}{2^6 3\pi^4} m_s s \right. \\ &\quad \left. - \frac{2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{2^6 3\pi^4} m_s s^2 + \frac{s^2}{2^9 3\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \right\}, \\ BB &= \int_{4m_s^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ -\frac{s^4}{2^9 5! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma G q \rangle + \langle \bar{s}g_s \sigma G s \rangle}{2^6 3\pi^4} m_s s \right. \\ &\quad \left. - \frac{2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle}{2^6 3\pi^4} m_s s^2 - \frac{s^2}{2^9 3\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \right\}. \end{aligned}$$

Differentiate the above sum rules with respect to the variable  $\frac{1}{M^2}$ , then eliminate the quantities  $f_{f_0(a_0)}^A$  and  $f_{f_0(a_0)}^B$ , we obtain

$$m_{f_0(a_0)}^{A2} = \int_{4m_s^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ \frac{s^5}{2^9 5! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma G q \rangle - \langle \bar{s}g_s \sigma G s \rangle}{2^6 3\pi^4} m_s s^2 \right. \\ \left. - \frac{2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{2^6 3\pi^4} m_s s^3 + \frac{s^3}{2^9 3\pi^4} \left\langle \frac{\alpha_s G G}{\pi} \right\rangle \right\} / AA, \quad (14)$$

$$m_{f_0(a_0)}^{B2} = \int_{4m_s^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ -\frac{s^5}{2^9 5! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma G q \rangle + \langle \bar{s}g_s \sigma G s \rangle}{2^6 3\pi^4} m_s s^2 \right. \\ \left. - \frac{2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle}{2^6 3\pi^4} m_s s^3 - \frac{s^3}{2^9 3\pi^4} \left\langle \frac{\alpha_s G G}{\pi} \right\rangle \right\} / BB. \quad (15)$$

It is easy to perform the  $s$  integral in Eqs.(12-15), we prefer this form for simplicity.

### 3 Numerical Results

The parameters are taken as  $\langle \bar{s}s \rangle = 0.8\langle \bar{u}u \rangle$ ,  $\langle \bar{s}g_s \sigma G s \rangle = 0.8\langle \bar{s}s \rangle$ ,  $\langle \bar{q}g_s \sigma G q \rangle = 0.8\langle \bar{q}q \rangle$ ,  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{q}q \rangle = (-219 MeV)^3$ ,  $\langle \frac{\alpha_s G G}{\pi} \rangle = (0.33 GeV)^4$ ,  $m_u = m_d = 0$  and  $m_s = 150 MeV$ . The main contributions to the sum rules come from the quark condensates terms, here we have taken the standard values and neglected the uncertainties, small variations of those condensates will not lead to larger changes about the numerical values. The threshold parameter  $s_0$  is chosen to vary between  $(1.4 - 1.6) GeV^2$  to avoid possible pollutions from higher resonances and continuum states. In the region  $M^2 = (1.2 - 3.2) GeV^2$ , the sum rules for  $m_{f_0}^A = m_{a_0}^A$  and  $m_{f_0}^B = m_{a_0}^B$  are almost independent of the Borel parameter  $M^2$  which are plotted in the Figure for  $s_0 = 1.5 GeV^2$  as an example. Due to the special quark constituents and Dirac structures of the interpolating currents, the scalar mesons  $f_0(980)$  and  $a_0(980)$  have degenerate masses. For the  $S^a - \bar{S}^a$  type interpolating currents  $J_{f_0}^A$  and  $J_{a_0}^A$ , the values for masses are about  $m_{f_0}^A = m_{a_0}^A = (0.96 - 1.02) GeV$ , while for the  $P^a - \bar{P}^a$  type interpolating currents  $J_{f_0}^B$  and  $J_{a_0}^B$ , the values for masses are about  $m_{f_0}^B = m_{a_0}^B = (0.95 - 1.01) GeV$ . Although the values for masses  $m_{f_0}^A = m_{a_0}^A$  lie a little above the masses  $m_{f_0}^B = m_{a_0}^B$ , we can not get to the conclusion that the scalar mesons  $f_0(980)$  and  $a_0(980)$  prefer the  $S^a - \bar{S}^a$  type interpolating currents  $J_{f_0}^A$  and  $J_{a_0}^A$  to the  $P^a - \bar{P}^a$  type interpolating currents  $J_{f_0}^B$  and  $J_{a_0}^B$ . Precise determination of what type interpolating currents we should choose calls for original theoretical approaches, the contributions from the direct instantons may do the work. In our recent work, we observe that the contributions from the direct instantons are considerable for the pentaquark state  $\Theta^+(1540)$  [14], furthermore, the contributions from the direct instantons can improve the QCD sum rule greatly in some channels, for example, the nonperturbative contributions from the direct instantons to the conventional operator product expansion can significantly improve the stability of chirally odd nucleon sum rules [15, 16]. Despite whatever the interpolating currents may be,

we observe that they both give the correct degenerate masses for the scalar mesons  $f_0(980)$  and  $a_0(980)$ , there must be some four-quark constituents in those mesons.

## 4 Conclusions

In this article, we take the point of view that the  $f_0(980)$  and  $a_0(980)$  mesons are four-quark states  $(qq)_3(\bar{q}\bar{q})_3$  in the ideal mixing limit, and devote to determine the values of their masses  $m_{f_0}$  and  $m_{a_0}$  in the framework of the QCD sum rules approach. Due to the special quark constituents and Dirac structures of the interpolating currents, the scalar mesons  $f_0(980)$  and  $a_0(980)$  have degenerate masses. For the  $S^a - \bar{S}^a$  type interpolating currents  $J_{f_0}^A$  and  $J_{a_0}^A$ , the values for masses are about  $m_{f_0}^A = m_{a_0}^A = (0.96 - 1.02)GeV$ , while for the  $P^a - \bar{P}^a$  type interpolating currents  $J_{f_0}^B$  and  $J_{a_0}^B$ , the values for masses are about  $m_{f_0}^B = m_{a_0}^B = (0.95 - 1.01)GeV$ . Although the values for masses  $m_{f_0}^A = m_{a_0}^A$  lie a little above the masses  $m_{f_0}^B = m_{a_0}^B$ , we can not get to the conclusion that the scalar mesons  $f_0(980)$  and  $a_0(980)$  prefer the  $S^a - \bar{S}^a$  type interpolating currents  $J_{f_0}^A$  and  $J_{a_0}^A$  to the  $P^a - \bar{P}^a$  type interpolating currents  $J_{f_0}^B$  and  $J_{a_0}^B$ . Despite whatever the interpolating currents may be, we observe that they both give the correct degenerate masses for the scalar mesons  $f_0(980)$  and  $a_0(980)$ , there must be some four-quark constituents in those mesons, our results support the four-quark model and the hybrid model. In the hybrid model, those mesons are four-quark states  $(qq)_3(\bar{q}\bar{q})_3$  in S-wave near the center, with some constituent  $q\bar{q}$  in P-wave, but further out they rearrange into  $(q\bar{q})_1(q\bar{q})_1$  states and finally as meson-meson states [5]. Precise determination of what type interpolating currents we should choose calls for original theoretical approaches, the contributions from the direct instantons may do the work.

## Acknowledgment

This work is supported by National Natural Science Foundation, Grant Number 10405009, and Key Program Foundation of NCEPU. The authors are indebted to Dr. J.He (IHEP) , Dr. X.B.Huang (PKU) and Dr. L.Li (GSCAS) for numerous help, without them, the work would not be finished. The author would also thanks Prof. M.Nielsen for helpful discussion.

## References

- [1] S. Godfray and J. Napolitano, Rev. Mod. Phys. **71** (1999) 1411.
- [2] R. L. Jaffe and K. Johnson, Phys. Lett. **B60** (1976) 201; R. L. Jaffe, Phys. Rev. **D15** (1977) 267, 281; **D17** (1978) 1444; N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315** (1989) 465; N. N. Achasov and V. V. Gubin, Phys. Rev. **D56**

- (1997) 4084; Phys. Rev. **D63** (2001) 094007; J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001.
- [3] J. Weinstein and N. Isgur, Phys. Rev. Lett. **48** (1982) 659; Phys. Rev. **D27** (1983) 588; Phys. Rev. **D41** (1990) 2236 ; F. E. Close, N. Isgur and S. Kumana, Nucl. Phys. **B389** (1993) 513; R. Kaminski, L. Lesniak and J. P. Maillet, Phys. Rev. **D50** (1994) 3145; N. N. Achasov, V. V. Gubin and V. I. Shevchenko, Phys. Rev. **D56** (1997) 203; Yu. S. Surovtsev, D. Krupa and M. Nagy, hep-ph/0311195.
- [4] N. A. Tornqvist, Z. Phys. **C68** (1995) 647; M. Boglione and M. R. Pennington, Phys. Rev. Lett **79** (1997) 1998; N. A. Tornqvist, hep-ph/0008136; N. A. Tornqvist and A. D. Polosa, Nucl. Phys. **A692** (2001) 259; A. Deandrea, R. Gatto, G. Nardulli, A. D. Polosa and N. A. Tornqvist, Phys. Lett. **B502** (2001) 79; F. De Fazio and M. R. Pennington, Phys. Lett. **B521** (2001) 15; M. Boglione and M. R. Pennington, Phys. Rev. **D65** (2002) 114010; Z. G. Wang, W. M. Yang and S. L. Wan, Eur. Phys. J. **C37** (2004) 223.
- [5] F. E. Close and N. A. Tornqvist, J. Phys. **G28** (2002) R249.
- [6] R. L. Jaffe, Phys. Rept. **409** (2005) 1; C. Amsler and N. A. Tornqvist, Phys. Rept. **389** (2004) 61.
- [7] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448.
- [8] L. Maiani, A. Polosa, F. Piccinini and V. Riquer, Phys. Rev. Lett. **93** (2004) 212002.
- [9] T. V. Brito, F. S. Navarra, M. Nielsen and M. E. Bracco, hep-ph/0411233.
- [10] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. **D12** (1975) 147 ; T. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, Phys. Rev. **D12** (1975) 2060 ; G. 't Hooft, Phys. Rev. **D14** (1976) 3432 [Erratum-ibid. Phys. Rev. **D18** (1978) 2199 ]; E. V. Shuryak, Nucl. Phys. **B203** (1982) 93 ; T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70** (1998) 323 ; E. Shuryak and I. Zahed, Phys. Lett. **B 589** (2004) 21.
- [11] J. I. Latorre and P. Pascual, J. Phys. **G11**, (1985) L231 .
- [12] S. Narison, Phys. Lett. **B175** (1986) 88; S. Narison, QCD Spectral Sum Rules, World Scientific Lecture Notes in Physics **26**.
- [13] R. T. Cahill, J. Praschifka and C. Burden Austral.J.Phys. **42** (1989) 161; M. Huang, hep-ph/0409167 .
- [14] Z. G. Wang, W. M. Yang and S. L. Wan, hep-ph/0501015.

- [15] A. E. Dorokhov and N. I. Kochelev , Z. Phys. **C46** (1990) 281.
- [16] M. Aw, M. K. Banerjee and H. Forkel, Phys. Lett. **B454** (1999) 147; H. Forkel and M. Nielsen, Phys. Rev. **D55** (1997) 1471; H. Forkel and M. K. Banerjee, Phys. Rev. Lett. **71** (1993) 484.





